QUESTION

Use Euler's theorem to find

(i) the units digit of 3^{250}.

(ii) the last two digits of 3^{250}.

ANSWER

The unit digit of an integer is given by its congruence class mod 10, and the final two digits by its congruence class mod 100.

Since 10=2.5 and $100 = 2^2.5^5$ we have $\phi(10) = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 4$ and $\phi(100) = 100\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 40$. Since gcd(3,10)=gcd(3,100)=1, we may use Eulers theorem to evaluate (i) 3^{250} mod 10 and (ii)3^{250} mod 100. (of course we could just do (ii) and deduce (i) from it- but doing both is useful for practice!)

(i) $3^{250} \equiv (3^4)^{62}.3^2 \equiv 1^{62}.3^2 \equiv 9$ mod 10. Thus the unit digit of 3^{250} is 9.

(ii) $3^{250} \equiv (3^{40})^6.3^{10} \equiv 1^6.3^{10} \equiv 3^{10}$ mod 100. Now $3^4 \equiv 81 \equiv -19$ mod 100, so $3^{10} \equiv (-19).(-19).9 \equiv 19.171 \equiv 19. - 29 \equiv -551 \equiv 49$ mod 100. Thus the last two digits of 3^{250} are 49.

[Comment: The above solution evaluates 3^{10} mod 100 by direct calculation. You might like to investigate the following rather quicker procedure:-

(a) Calculate 3^{10} mod 4. (b) Calculate 3^{10} mod 25. (c) Use the Chinese Remainder theorem to find a unique simultaneous solution of the congruences (a) and (b) mod 100.]