QUESTION
Find $\phi(432)$ and hence find the value of 5^{290} mod 432.
ANSWER
$432 = 4.108 = 4.4.27 = 2^4.3^3$. Thus $\phi(432) = 2^4.3^3\left(1-\frac{1}{2}\right)\left(1-\frac{1}{2}\right) = 2^4.3^3.\frac{1}{2}.\frac{2}{3} = 2^4.3^2 = 144$. Now gcd(5,432)=1, so by Eulers Theorem (th.5.1), $5^{144} \equiv 1$ mod 432. Thus $5^{290} = (5^{144})^2.5^2 \equiv 1^2.5^2 \equiv 25$ mod 432.