

**Question**

Show that if all the elements of one column of a  $3 \times 3$  determinant are multiplied by a constant  $k$ , then the value of the determinant is multiplied by  $k$ . Give a geometrical interpretation in terms of volumes.

**Answer**

Let my determinant be  $\Delta$  where

$$\begin{aligned}\Delta &= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\ &= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}\end{aligned}$$

Now multiply 1st row by  $k$

$$\begin{aligned}\Delta_1 &= \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \\ &= ka_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - kb_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + kc_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \\ &= k \left\{ a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \right\} \\ &= k \Delta \text{ as required}\end{aligned}$$