

Question

Prove the rule that a determinant vanishes when two rows are identical.

Answer

Let my determinant be Δ where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ by rule that interchanging 2 rows means } \Delta \rightarrow -\Delta \text{ and}$$

interchanging first 2 rows

$$= - \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\Delta$$

Thus

$$\Delta = -\Delta$$

$$\Rightarrow 2\Delta = 0$$

$$\Rightarrow \underline{\Delta = 0}$$

Hence 2 rows identical $\Rightarrow \Delta = 0$.