Vector Fields
Conservative Fields

Question
For the following vector field, find whether it is conservative. If so, find a corresponding potential
$\underline{F}(x,y,z) = ex^2 + y^2 + z^2(xz\underline{i} + yz\underline{j} + xy\underline{k})$
Answer

$$\begin{aligned} F_1 &= xze^{x^2+y^2+z^2} \\ F_2 &= yze^{x^2+y^2+z^2} \\ F_3 &= xye^{x^2+y^2+z^2} \end{aligned}$$

This gives

$$\begin{aligned} \frac{\partial F_1}{\partial y} &= 2xyze^{x^2+y^2+z^2} = \frac{\partial F_2}{\partial x} \\ \frac{\partial F_1}{\partial x} &= (x + 2xz^2)e^{x^2+y^2+z^2} \\ \frac{\partial F_3}{\partial x} &= (y + 2x^2y)e^{x^2+y^2+z^2} \neq \frac{\partial F_1}{\partial z}. \end{aligned}$$

So $\underline{F}$ cannot be conservative.