QUESTION

**(a)** Four projects are being considered for execution over the next three years. The expected returns for each project, yearly expenditures and the maximum fund available each year (in millions of pounds) are given in the following table.

| | Expenditure for year | | | |
|---|---|---|---|---|
| Projects | Year 1 | Year 2 | Year 3 | Returns |
| 1 | 3 | 4 | 2 | 20 |
| 2 | 4 | 3 | 2 | 20 |
| 3 | 4 | 3 | 3 | 30 |
| 4 | 3 | 2 | 5 | 30 |
| Maximum funds | 10 | 11 | 12 | |

Assume that each approved project will be executed over the 3-year period. The objective is to select projects that maximize the total return. Give an integer programming formulation for this problem.

**(b)** A company supplies 10 retail outlets, with Outlet $j$ requiring $d_j$ units monthly. The company can rent storage facilities in up to 5 warehouses, with Warehouse $i$ having a storage capacity of $s_i$ units and a monthly rent fee of $r_i$. There is a cost of $c_{ij}$ to ship one unit from Warehouse $i$ to Outlet $j$. Let $x_{ij}$ be the number of units shipped monthly from Warehouse $i$ to Outlet $j$, and $y_i = 1$ if Warehouse $i$ is used and $y_i = 0$ otherwise. Give a mixed integer programming formulation for the minimization of the total cost.

**(c)** Solve the following integer programming problem by a branch and bound algorithm.

$$\begin{array}{ll} \text{Maximize} & z = 3x_1 + x_2 + 4x_3 \\ \text{subject to} & 6x_1 + 3x_2 + 5x_3 \leq 25 \\ & 3x_1 + 4x_2 + 5x_3 \leq 20 \\ & x_i \geq 0 \text{ and integer, for } i = 1, 2, 3. \end{array}$$

ANSWER

**(a)** The integer programming formulation is given by

$$\begin{array}{ll} \text{maximize} & z = 20x_1 + 20x_2 + 30x_3 + 30x_4 \\ \text{subject to} & 3x_1 + 4x_2 + 4x_3 + 3x_4 \leq 10 \\ & 4x_1 + 3x_2 + 3x_3 + 2x_4 \leq 11 \\ & 2x_1 + 2x_2 + 3x_3 + 5x_4 \leq 12 \\ & 0 \leq x_i \leq 1 \text{ and integer} \end{array}$$

**(b)** We have the following constraints:

1. For the capacity of warehouse $i$

$$\sum_{j=1}^{10} x_{ij} \leq s_i \; i = 1, 2, \ldots, 5$$

2. For the demand at outlet $j$:

$$\sum_{i=1}^{5} x_{ij} = d_j \; j = 1, 2, \ldots, 10$$

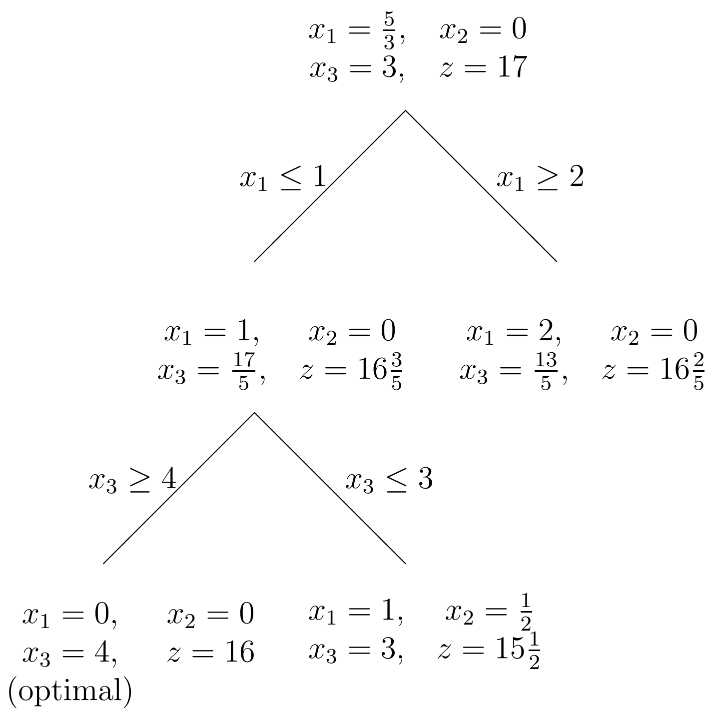
3. For variable $y_i$:
   $y_i = 1$ if Warehouse $i$ is used or any one of $x_{ij}$ is positive. Therefore we may set

$$y_i \geq \frac{\sum_{j=1}^{10} x_{ij}}{s_i}.$$

The integer programming formulation is given by

| | |
|---|---|
| minimmize | $z = \sum_{i=1}^{5} \sum_{j=1}^{10} c_{ij} x_{ij} + \sum_{i=1}^{5} r_i y_i$ |
| subject to | $\sum_{j=1}^{10} x_{ij} \leq s_i \; i = 1, 2, \ldots, 5$ |
| | $\sum_{i=1}^{5} x_{ij} = d_j \; j = 1, 2, \ldots, 10$ |
| | $y_i \geq \frac{\sum_{j=1}^{10} x_{ij}}{s_i}, \; i = 1, 2, \ldots, 5$ |
| | $x_{ij} \geq 0, \; (0 \leq y_i \leq 1$ and integer). |

**(c)**


$x_1 = \frac{5}{3}$, $x_2 = 0$
$x_3 = 3$, $z = 17$
$x_1 \leq 1$
$x_1 \geq 2$
$x_1 = 1$, $x_2 = 0$
$x_3 = \frac{17}{5}$, $z = 16\frac{3}{5}$
$x_1 = 2$, $x_2 = 0$
$x_3 = \frac{13}{5}$, $z = 16\frac{2}{5}$
$x_3 \geq 4$
$x_3 \leq 3$
$x_1 = 0$, $x_2 = 0$
$x_3 = 4$, $z = 16$
(optimal)
$x_1 = 1$, $x_2 = \frac{1}{2}$
$x_3 = 3$, $z = 15\frac{1}{2}$