QUESTION

Show that the following linear programming problem can be formulated as a minimum cost network flow problem.

Minimize	$z = 5x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5$
	$+5x_6 + 5x_7 + 3x_8 + 6x_9 + 5x_{10}$
subject to	$x_1, \ldots, x_{10} \geq 0$
	$x_1 + x_2 = 12$
	$x_2 + x_3 + x_4 = 10$
	$x_4 + x_5 = 13$
	$x_5 + x_6 = 16$
	$x_7 + x_8 = 6$
	$x_8 + x_9 \leq 9$
	$x_9 + x_{10} \geq 8$
	$x_3 + x_7 + x_{10} = 20.$

Starting with a solution in which x_1, x_2, x_5, x_6, x_8 and x_{10} take positive values, and the constraints $x_8 + x_9 \leq 9$ and $x_9 + x_{10} \geq 8$ are satisfied as strict inequalities, use the network simplex method to solve the problem.

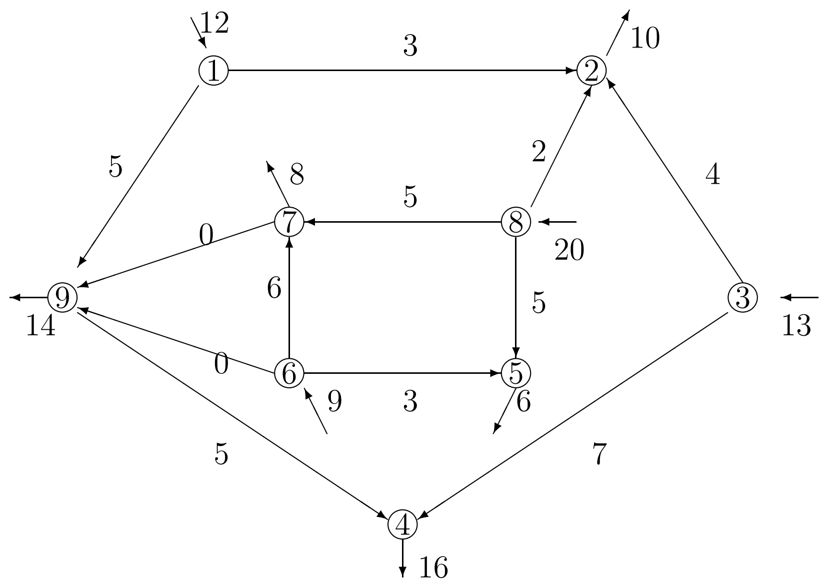
ANSWER

Adding slack variables and multiplying some constraints by -1, the formulation is

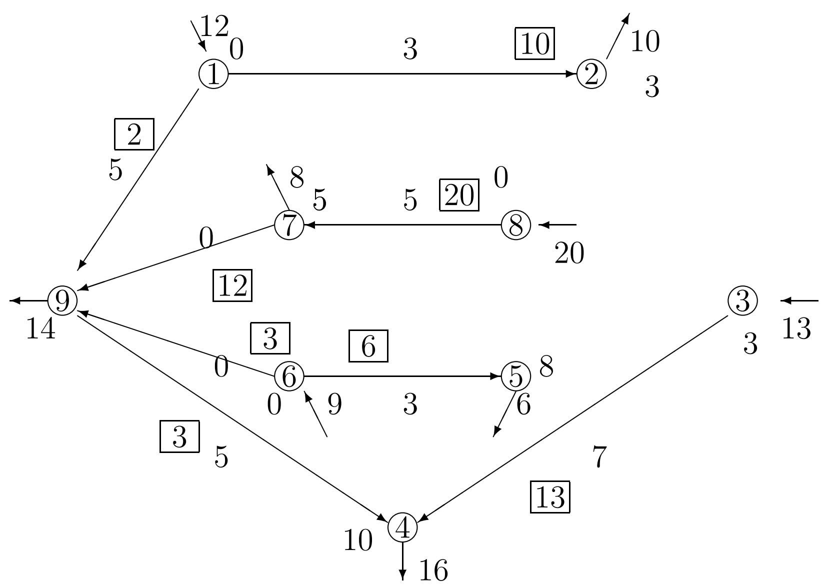
	Minimize	$z = 5x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 + 5x_6 + 5x_7 + 3x_8 + 6x_9 + 5x_{10}$
	subject to	$x_i \geq 0 \; i = 1, \ldots, 10$
(1)		$x_1 + x_2 = 12$
(2)		$-x_2 - x_3 - x_4 = -10$
(3)		$x_4 + x_5 = 13$
(4)		$-x_5 - x_6 = -16$
(5)		$-x_7 - x_8 = -6$
(6)		$x_8 + x_9 + s_1$
(7)		$-x_9 - x_{10} + s_2 = -8$
(8)		$x_3 + x_9 + x_10 = 10$
(9)		$-x_1 + x_6 - s_1 - s_2 = -14$

(where the last redundant constraint is obtained by summing the others).

The network is

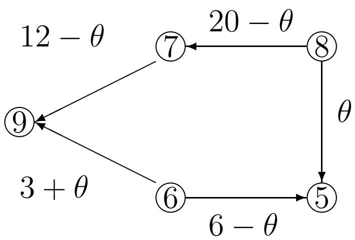


The initial tree solution is

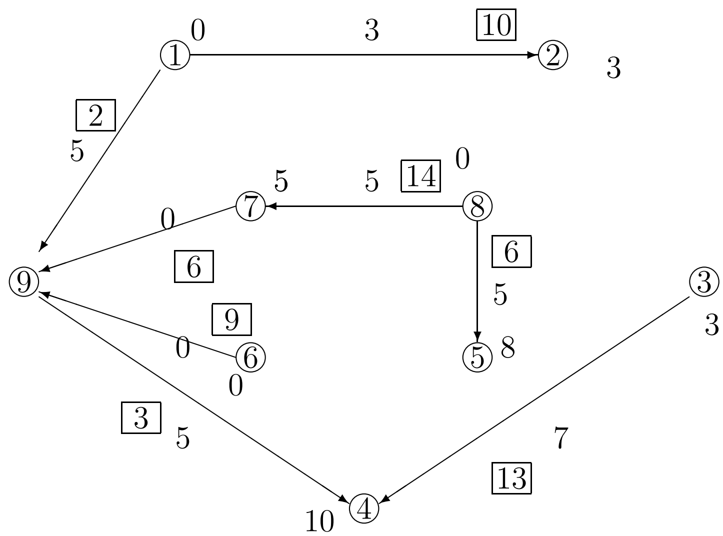


Non-basic	$y_i + c_{ij} - y_j$
(3,2)	4
(8,2)	-1
(8,5)	-3
(6,7)	6

Entering arc is (8,5)

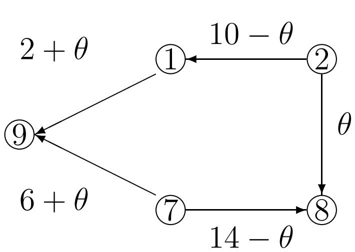


$\theta = 6$
Leaving arc is (6,5)

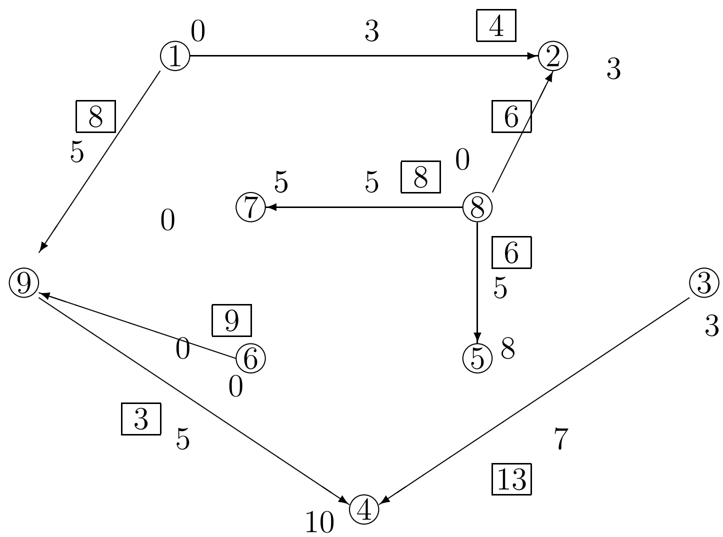


Non-basic	$y_i + c_{ij} - y_j$
(3,2)	4
(8,2)	-1
(6,5)	3
(6,7)	6

Entering arc is (8,2)



Leaving arc is (9,7)



Non-basic	$y_i + c_{ij} - y_j$
(3,2)	4
(6,5)	2
(6,7)	5
(9,7)	1

Thus, we have an optimal solution

$x_1 = 8$ $x_2 = 4$ $x_3 = 6$ $x_4 = 0$ $x_5 = 13$ $x_6 = 3$ $x_7 = 6$ $x_8 = 0$ $x_9 = 0$ $x_{10} = 8$ $z = 240$