QUESTION

For each of the following matrices find the kernel and the image of the corresponding linear transformation τ.

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -4 \\ 1 & 0 & -4 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & -6 \\ 34 & -17 & 51 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 5 & 5 & 5 & 5 \\ 3 & 1 & -1 & -3 \end{bmatrix}$$

ANSWER

The first matrix has rank 2 and nullity 1. the vector (4,-4,1) is a basis for $\ker\tau$, so $\ker\tau$ is the line $x = -y = 4z$. Choosing two independent vectors not on this line, such as (1,0,0) and (0,1,0), the image imτ has as a basis the images of these two vectors. So imτ is spanned by (-1,0,1) and (-1,-1,0) which generate the plane $x - y + z = 0$.

The second matrix has rank 1 and nullity 2. The kernel is the plane $2x - y + 3z = 0$. The vector (1,0,0) is not on this plane, it maps to (2,-4,34), so the image is the line $x = -\frac{y}{2} = \frac{z}{17}$

The third matrix has rank 2 and nullity 2. Gaussian elimination shows that the kernel is the plane specified by $w = y + 2z$ and $x = -2y - 3z$. Two independent vectors not in this plane are (1,0,0,0) and (0,1,0,0) which map to (1,4,5,3) and (2,3,5,1) respectively. Gaussian elimination on these shows that they span the plane specified by $w = -y + z$ and $x = -y - z$.