QUESTION
Let f be an entire function such that $|f(z)| \leq A|z|$ for all z, where A is a fixed positive number. Show that $f(z) = az$, where a is a complex constant. (Hint: Use the Cauchy inequalities to show that that the second derivative of f is everywhere zero.)
ANSWER
If D denotes the open unit disc, then if $f(\mathbf{C}) = D$, we have $|f(z)| < 1$, for all $z \in \mathbf{C}$, so that f is a bounded analytic function and hence by Liouville's Theorem, f is constant and hence f cannot map $\mathbf{C}$ *onto* D.