QUESTION

Find the real and imaginary parts of $\sin(1 + i)$.

ANSWER

$f(z)$ analytic implies that $\partial u/\partial x = \partial v/\partial y$.

$\overline{f(z)}$ analytic implies that $\partial u/\partial x = -\partial v/\partial y$. Thus $\partial u/\partial x = \partial v/\partial y = 0$, and also

$\partial u/\partial y = \partial v/\partial x = 0$. Thus u and v are constants, (See Theorem 3.4) and so f is constant. Now suppose that f is constant and that $|f|$ is constant. Then $|f^2| = f\overline{f}$ is constant. We deduce that $\overline{f}$ is constant so by the first part f is constant.