Multiple Integration
Iteration of Double Integrals

Question
The functions $f(x,t)$ and $f_1(x,t)$ are continuous on the rectangle $a \le x \le b$, $c \le t \le d$.

$$\begin{aligned} g(x) &= \int_c^d f(x,t)\,dt \\ G(x) &= \int_c^d f_1(x,t)\,dt \end{aligned}$$

Show that $g'(x) = G(x)$ if $a < x < b$.
[Hint: evaluate $\int_a^x G(u)\,du$ by reversing the order of iteration. Then differentiate the result.]
Answer

$$\begin{aligned} \int_a^x G(u)\,du &= \int_a^x du \int_c^d f_1(u,t)dt \\ &= \int_c^d dt \int_a^x f_1(u,t)\,du \\ &= \int_c^d (f(x,t) - f(a,t))\,dt \\ &= g(x) - C \end{aligned}$$

With $C = \displaystyle\int_c^d f(a,t)\,dt$ being independent of x. If we apply the fundamental theorem of Calculus then it can be seen that

$$g'(x) = \frac{d}{dx}\int_a^x G(u)\,du = G(x)$$