

Multiple Integration
Iteration of Double Integrals

Question

Find the volume for the solid defined by

The space over the triangle defined by the vertices $(0, 0)$, $(a, 0)$ and $(0, b)$, and below the plane $z = 2 - (x/a) - (y/b)$.

Answer

$$\begin{aligned} V &= \iint_Y \left(2 - \frac{x}{a} - \frac{y}{b} \right) dA \\ &= \int_0^{b(1-(x/a))} \left(2 - \frac{x}{a} - \frac{y}{b} \right) dy \\ &= \int_0^a \left[\left(2 - \frac{x}{a} \right) b \left(1 - \frac{x}{a} \right) - \frac{1}{2b} b^2 \left(1 - \frac{x}{a} \right)^2 \right] dx \\ &= \frac{b}{a} \int_0^a \left(3 - \frac{4x}{a} + \frac{x^2}{a^2} \right) dx \\ &= \frac{b}{2} \left(3x - \frac{2x^2}{a} + \frac{x^3}{3a^2} \right) \Big|_0^a \\ &= \frac{2}{3} abc \text{ u. units} \end{aligned}$$