

**Multiple Integration**  
*Iteration of Double Integrals*

**Question**

Find the volume of the given solid

Below  $z = x^2 \sin(y^4)$  and over the triangle in the  $xy$ -plane defined by the vertices  $(0, 0)$ ,  $(0, \pi^{1/4})$  and  $(\pi^{1/4}, \pi^{1/4})$ .

**Answer**

$$\begin{aligned} V &= \int_0^{\pi^{1/4}} dy \int_0^y x^2 \sin(y^4) dx \\ &= \frac{1}{3} \int_0^{\pi^{1/4}} y^3 \sin(y^4) dy \end{aligned}$$

$$\begin{aligned} \text{Let } u &= y^4 \\ du &= 4y^3 dy \end{aligned}$$

$$\begin{aligned} \Rightarrow V &= \frac{1}{12} \int_0^{\pi} \sin u du \\ &= \frac{1}{6} \text{cu. units} \end{aligned}$$