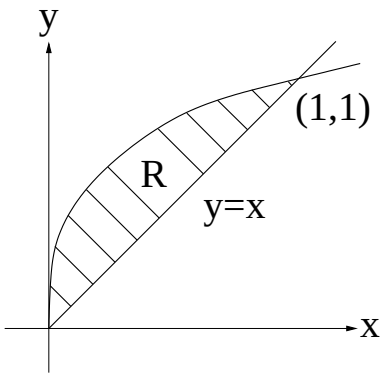
Multiple Integration
Iteration of Double Integrals

Question
Sketch the domain of integration, and calculate the iterated integral for

$$\int_0^1 dx \int_x^{x^{1/3}} \sqrt{1-y^4}\, dy$$

Answer



$$\begin{aligned} I &= \int_0^1 dx \int_x^{x^{1/3}} \sqrt{1-y^4}\, dy \\ &= \iint_R \sqrt{1-y^4}\, dA \\ &= \int_0^1 y\sqrt{1-y^4}\, dy - \int_0^1 y^3\sqrt{1-y^4}\, dy \end{aligned}$$

$$\text{Let } u = y^2 \qquad \text{Let } v = 1-y^4$$
$$\Rightarrow du = 2ydy \qquad \Rightarrow dv = -4y^3 dy$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{2}\int_0^1 \sqrt{1-u^2}\, du + \frac{1}{4}\int_1^0 v^{1/2}\, dv \\ &= \frac{1}{2}\left(\frac{\pi}{4}\times 1^2\right) + \frac{1}{6}v^{3/2}\Big|_1^0 \\ &= \frac{\pi}{8} - \frac{1}{6} \end{aligned}$$