

Multiple Integration
Iteration of Double Integrals

Question

Calculate the given double integrals by iteration

$$\iint_R \frac{x}{y} e^y dA$$

With R being the region $0 \leq x \leq 1$, $x^2 \leq y \leq e^x$.

Answer

$$\begin{aligned} I &= \iint_R \frac{x}{y} e^y dA \\ &= \int_0^1 \frac{e^y}{y} dy \int_y^{\sqrt{y}} x dx \\ &= \frac{1}{2} \int_0^1 (1 - y^y) dy \end{aligned}$$

$$U = 1 - y \quad dV = e^y dy$$

$$dU = -dy \quad V = e^y$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{2} \left[(1 - y)e^y \Big|_0^1 + \int_0^1 e^y dy \right] \\ &= -\frac{1}{2} + \frac{1}{2}(e - 1) = \frac{e}{2} - 1 \end{aligned}$$