

**Multiple Integration**  
***Iteration of Double Integrals***

**Question**

Calculate the given double integral by iteration in the region defined by the given curves.

$$\iint_D \ln x \, dA$$

With  $D$  being the region bounded by  $2x + 2y = 5$  and  $xy = 1$  in the first quadrant.

**Answer**

For intersection:  $xy = 1, 2x + 2y = 5$ .

$\Rightarrow 2x^2 - 5x + 2 = 0$ , or  $(2x - 1)(x - 2) = 0$ . And so the intersections are at  $x = 1/2$  and  $x = 2$ .

$$\begin{aligned} I &= \iint_D \ln x \, dA = \int_{1/2}^2 \ln x \, dx \int_{1/x}^{(5/2)-x} dy \\ &= \int_{1/2}^2 \ln x \left( \frac{5}{2} - x - \frac{1}{x} \right) dx \\ &= \int_{1/2}^2 \ln x \left( \frac{5}{2} - x \right) dx - \frac{1}{2} (\ln x)^2 \Big|_{1/2}^2 \end{aligned}$$

$$\begin{aligned} U &= \ln x & dV &= \left( \frac{5}{2} - x \right) dx \\ dU &= \frac{dx}{x} & V &= \frac{5}{2}x - \frac{x^2}{2} \end{aligned}$$

$$\begin{aligned} I &= -\frac{1}{2} \left( (\ln 2)^2 - \left( \ln \frac{1}{2} \right)^2 \right) + \left( \frac{5}{2}x - \frac{x^2}{2} \right) \ln x \Big|_{1/2}^2 \\ &\quad - \int_{1/2}^2 \left( \frac{5}{2} - \frac{x}{2} \right) dx \\ &= (5 - 2) \ln 2 - \left( \frac{5}{4} - \frac{1}{8} \right) \ln \frac{1}{2} - \frac{15}{4} + \frac{15}{16} \\ &= \frac{33}{8} \ln 2 - \frac{45}{16} \end{aligned}$$