Multiple Integration
Iteration of Double Integrals

Question
Find the volume of the given solid
Below $z = 1 - y^2$ and over $z = x^2$.
Answer
$z = 1 - y^2$ and $z = x^2$ interesct on the cylinder $x^2 + y^2 = 1$, and so the volume can be given by

$$\begin{aligned}
V &= \iint_{x^2+y^2\leq 1} (1 - y^2 - x^2)\, dA \\
&= 4\int_0^1 dx \int_0^{\sqrt{1-x^2}} (1 - x^2 - y^2)\, dy \\
&= 4\int_0^1 dx \left((1-x^2)y - \frac{y^3}{3}\right)\Big|_{y=0}^{y=\sqrt{1-x^2}} \\
&= \frac{8}{3}\int_0^1 (1-x^2)^{3/2}\, dx \\
\text{Let } x &= \sin u \\
du &= \cos u du
\end{aligned}$$

$$\begin{aligned}
\Rightarrow V &= \frac{8}{3}\int_0^{\pi/2} \cos^4 u\, du \\
&= \frac{2}{3}\int_0^{\pi/2} (1+\cos 2u)^2\, du \\
&= \frac{2}{3}\int_0^{\pi/2} \left(1 + 2\cos 2u + \frac{1+\cos 4u}{2}\right) du \\
&= \frac{2}{3}\frac{3}{2}\frac{\pi}{2} = \frac{\pi}{2}\text{cu. units}
\end{aligned}$$