**Question**

Show that the functions (a) $x^2 - y^2 + 2y$ and (b) $\sin x \cosh y$ are harmonic in any finite region of the $z$-plane where $z = x + iy$.

**Answer**

**(a)** If $\phi = x^2 - y^2 + 2y$

$$\frac{\partial^2 \phi}{\partial x^2} = 2, \ \frac{\partial^2 \phi}{\partial y^2} = -2 \Rightarrow \bigtriangledown^2 phi = 0$$

$\Rightarrow \phi$ harmonic in $\mathbf{R}$.

**(b)** If $\phi = \sin x \cosh y$

$$\frac{\partial^2 \phi}{\partial x^2} = -\sin x \cosh y, \ \frac{\partial^2 \phi}{\partial y^2} = \sin x \cosh y \Rightarrow \bigtriangledown^2 \phi = 0$$

$\Rightarrow \phi$ harmonic in $\mathbf{R}$.