Question

An object falls under gravity near the surface of the earth, and its motion is impeded by air resistance proportional to its speed. The equation of motion for the object is the second order differential equation

$$\frac{d^2y}{dx^2} = -g - k\frac{dy}{dx}$$

where k and g are positive constants.

(i) Show by letting $v = \frac{dy}{dt}$ we can obtain the first order differential equation

$$\frac{dv}{dt} + kt = -g$$

and hence using an integrating factor find the velocity of the object as a function of time, given that at $t = 0$ it was initially at speed $v = v_0$.

(ii) Find the limiting velocity $lim_{t\to\infty}v(t)$.

(iii) If the object was initally at height y_0, find its height $y(t)$ at time t.

Answer