QUESTION

Use Laplace transforms to solve the simultaneous equations

$$\begin{aligned} y' + y + z' &= 0 \\ y' - y + 2z' &= e^{-x} \end{aligned}$$

given that $y = \frac{1}{2}$ and $z = 0$ at $x = 0$.

ANSWER

$$\left.\begin{aligned} y' + y + z' = 0 \\ y' - y + 2z' = e^{-x} \end{aligned}\right\} \quad y(0) = \tfrac{1}{2}, \quad z(0) = 0$$

$$\begin{aligned} &sY(s) - y(0) + Y(s) + sZ(s) - z(0) = 0 \\ &sY(s) - y(0) - Y(s) + 2(sZ(s) - z(0)) = \tfrac{1}{s+1} \end{aligned}$$

$$(s+1)Y + sZ = \frac{1}{2} \qquad (1)$$

$$(s-1)Y + 2sZ = \frac{1}{2} + \frac{1}{s+1} \qquad (2)$$

$2 \times (1) - (2) \Rightarrow (s+3)Y = \frac{1}{2} - \frac{1}{s+1}$

$$\begin{aligned} Y &= \frac{1}{2(s+3)} - \frac{1}{(s+1)(s+3)} \\ &= \frac{1}{s+3} - \frac{1}{2}\frac{1}{s+1} \\ &\Rightarrow y = e^{-3x} - \frac{1}{2}e^{-x} \\ &\Rightarrow Z = \frac{1}{2s} - \frac{s+1}{s}Y \\ &= \frac{1}{2s} - \frac{s+1}{s(s+3)} + \frac{1}{2s} \\ &= \frac{1}{s} - \frac{1}{s+3} - \frac{1}{s(s+3)} \\ &= \frac{2}{3}\frac{1}{s+3} \\ &\Rightarrow z = \frac{2}{3} = \frac{2}{3}e^{-3x} \end{aligned}$$