

QUESTION

For each of the following pairs (a, b) , use the Euclidean algorithm to find $\gcd(a, b)$ and write it in the form $ax + by$ for suitable integers x and y .

(i) $(12, 57)$.

(ii) $(743, 601)$.

ANSWER

(i)

$$57 = 12 \cdot 4 + 9$$

$$12 = 9 \cdot 1 + 3$$

$$9 = 3 \cdot 3 + 0$$

so $\gcd(57, 12) = \text{last non-zero remainder} = 3$.

Substituting back,

$$\begin{aligned} 3 &= 12 - 9 \cdot 1 \\ &= 12 - (57 - 12 \cdot 4) \\ &= 12 \cdot 5 - 57 \cdot 1 \end{aligned}$$

so $x = 5$, $y = -1$ is a solution.

(ii)

$$743 = 601 \cdot 1 + 142$$

$$601 = 142 \cdot 4 + 33$$

$$142 = 33 \cdot 4 + 10$$

$$33 = 10 \cdot 3 + 3$$

$$10 = 3 \cdot 3 + 1$$

$$3 = 3 \cdot 1 + 0$$

so $\gcd(743, 601) = \text{last non-zero remainder} = 1$.

Substituting back

$$\begin{aligned} 1 &= 10 - 3.3 \\ &= 10 - (33 - 10.3).3 \\ &= 10.10 - 33.3 \\ &= (142 - 33.4).10 - 33.3 \\ &= 142.10 - 33.43 \\ &= 142.10 - (601 - 142.4).43 \\ &= 142.182 - 601.43 \\ &= (743 - 601.1).182 - 601.43 \\ &= 743.182 - 601.225 \end{aligned}$$

so $x = 182$, $y = -225$ is a solution.