QUESTION

Prove that $\gcd(n^3 + n - 1, n + 1)$ is either 1 or 3. Can both possibilities occur?

ANSWER

By long division, $n^3 + n - 1 = (n^2 - n + 2)(n + 1) - 3$, so by lemma 1.9 $\gcd(n^3 + n - 1, n + 1) = \gcd(n + 1, 3)$ and as the only divisors of 3 are 1 and 3, this nust be 1 or 3.

Both possibilities occur: e.g. $n = 1$ gives gcd=1 and $n = 2$ gives gcd=3.