QUESTION

Prove, by induction, that for all $n \geq 1,\ 5^{2n}+7$ is divisible by 8.

ANSWER

Take as inductive hypothesis the statement $8|5^{2n}+7$.

When $n=1,\ 5^{2n}+7=32=8.4$, so induction begins.

Now $8|5^{2n}+7$ implies $5^{2n}+7=8k$ for some integer k, so $5^{2n}=8k-7$.

Thus $5^{2(n+1)}+7=5^{2n}.5^2+7=25(8k-7)+7=25.8k-7.24=8(25k-21)$.

Thus $5^{2(n+1)}+7$ is divisible by 8.

This completes the inductive step, so $8|5^{2n}+7$ is true for all natural numbers N by induction.