

Question

For each of the following differential equations sketch:

- i) Several isoclines
- ii) The direction field
- iii) Several solution curves

1. $\frac{dy}{dx} = \frac{x}{y}$

2. $\frac{dy}{dx} = -\frac{x}{y}$ (*)

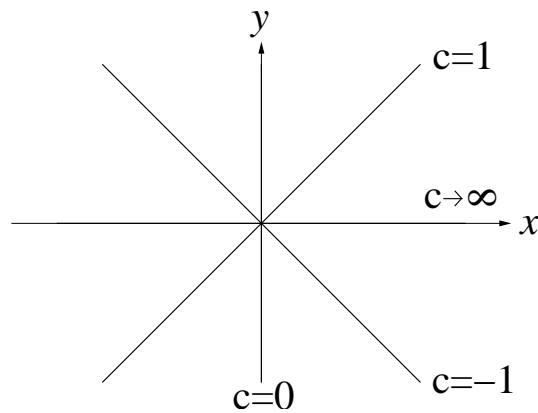
3. $\frac{dy}{dx} = xy$

4. $\frac{dy}{dx} = x^2 + y$ (*)

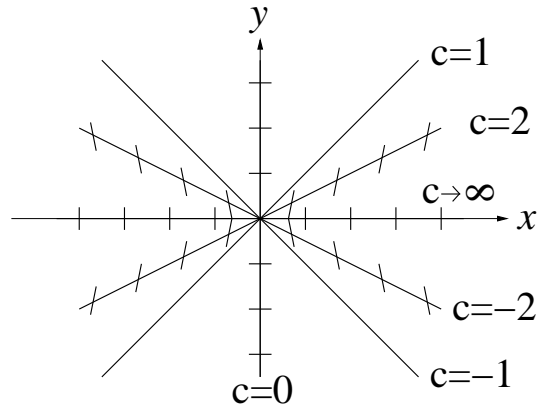
Answer

1. **Isoclines:** Curves where $\frac{x}{y} = C = \text{constant}$,

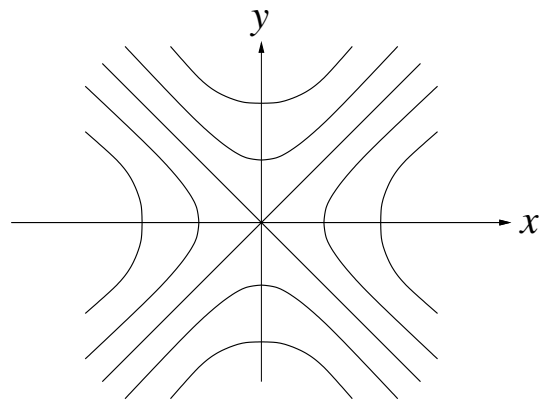
i.e. $x = cy$.



Direction field: On $x = cy$, $\frac{dy}{dx} = c$

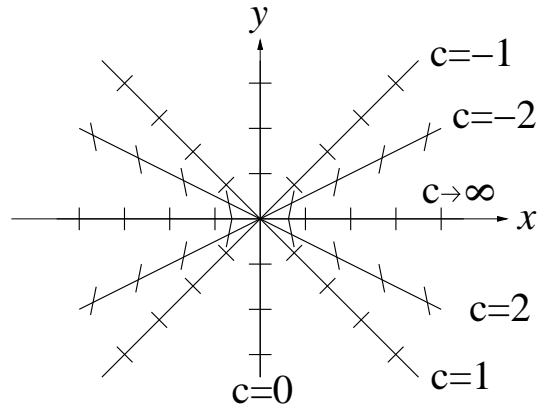


Solution curves: (connect direction field lines)

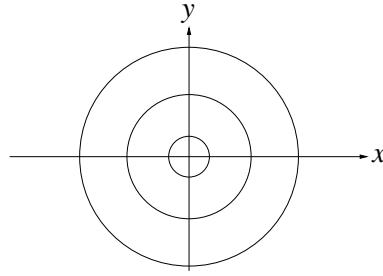


2. **Isoclines** are $-\frac{x}{y} = C, \Rightarrow y = -\frac{1}{c}x$

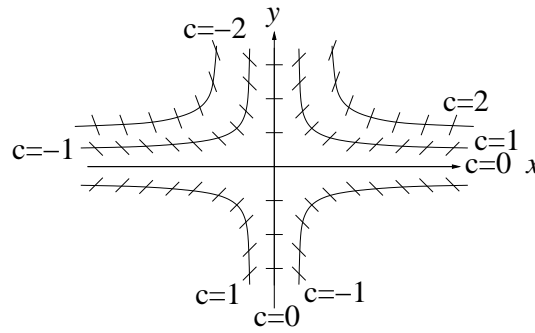
Direction field on $y = -\frac{1}{c}x, \frac{dy}{dx} = c$



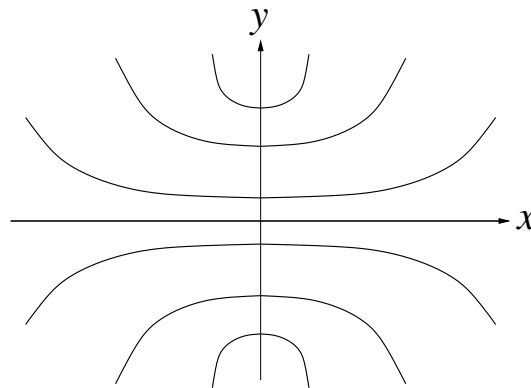
Solution curves



3. **Isoclines** are $xy = c$, (hyperbola) $\Rightarrow y = -\frac{c}{x}$
Direction field on $y = \frac{c}{x}$, draw small line with slope c .

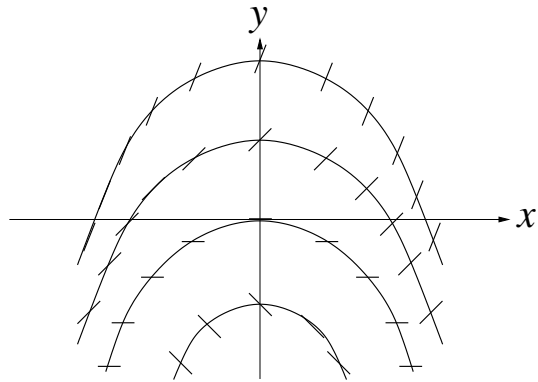


Solution curves



4. **Isoclines** are $x^2 + y = c$, (parabola) $\Rightarrow y = -x^2 + c$

Direction field draw short lines of slope c on the curves $y = -x^2 + c$.



Solution curves, join together direction fields

