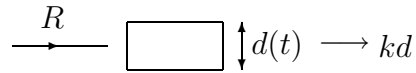


Question

Water flows into a reservoir from a river at the rate of $R \text{ kgs}^{-1}$. The outflow from the reservoir is $kd \text{ kgs}^{-1}$, where d is the depth of the reservoir and $k(> 0)$ is a constant. Assuming that the reservoir is a cylinder with cross-sectional area A and depth d find the depth of the reservoir as a function of time if it initially has a depth d_0 .

Answer

Mass is conserved, so $\frac{dm}{dt} = \text{net inflow}$; $m = Ad\rho$

$$A\rho\dot{d} = R - kd$$

This differential equation for d has the solution:

$$d(t) = \frac{R}{k} - B \exp\left(-\frac{k}{A\rho}t\right)$$

Now $d(0) = d_0$ therefore $d = \frac{R}{k} + \left(d_0 - \frac{R}{k}\right) \exp\left(-\frac{kt}{A\rho}\right)$