QUESTION

Use part (i) of question 4 above to prove that if p is a prime greater than 3, then $p^2 + 2$ is composite.

ANSWER

If p is a prime > 3, then by question 4(i), either p = 6k + 1 or p = 6k + 5 for some integer k. Thus $p^2 + 2$ is either $(6k + 1)^2 + 2 = 36k^2 + 12k + 3$ or $(6k + 1)^2 + 2 = 36k^2 + 60k + 27$ and both of these are divisible by 3. As $p^2 + 2 > p > 3$, $p^2 + 2$ must be composite.