QUESTION

Solve the following five-period production planning problem for which no shortages must occur. The initial stock level is zero. The cost of a set-up in any period is £100, and the holding cost is £2 per unit of stock held at the end of each period. Determine an optimal production plan if demnd is as follows.

Period	1	2	3	4	5
Demand	40	20	10	30	20

If the stock level at the end of period 5 is required to be 40, what production plan would you use?

ANSWER

This is the Wagner-Whitin dynamic model with $h = 2,\ s = 100$ and $N = 5$.

DIAGRAM

$$
\begin{aligned}
l_0 &= 0 \\
l_1 &= 0 + 100 = 100 \\
l_2 &= min\{0 + 140, 100 + 100\} = 140 \\
l_3 &= min\{0 + 180, 100 + 120, 140 + 100\} = 180 \\
l_4 &= min\{0 + 360, 100 + 240, 140 + 160, 180 + 100\} = 280 \\
l_5 &= min\{0 + 520, 100 + 360, 140 + 240, 180_1 40, 280 + 100\} = 320
\end{aligned}
$$

The shortest path in the network is 0 - 3 - 5. The production plan is

Period	1	2	3	4	5
Production	70	0	0	50	0

Cost=£320.

In the final stock is required to be 40, the demand in period 5 is effectively 60. Costs are as in the network above except the costs of arcs entering node 5 become 840, 600, 400, 220 and 100. We obtain l_1, l_2, l_3 and l_4 as before.

$$l_5 = min\{0 + 840, 100 + 600, 140 + 400, 180 + 220, 280 + 100\} = 380$$

The shortest path in the network is 0 - 3 - 4 - 5. The production plan is

Period	1	2	3	4	5
Production	10	0	0	30	60

Cost=£380 (excluding the holding cost at the end of period 5).