QUESTION

The stock holding cost for a product is £45 per item per annum, and the cost of placing an order for a replenishment is £200. Demand is steady and the annual demand is 2000 items. Shortages must not occur. The purchase cost depends on the number of items Q in the order: the cost per item is £c, where

$$c = \begin{cases} 50 - Q/100 & \text{for } Q \leq 500; \\ 45 & \text{for } 500 \leq Q < 1000; \\ 40 & \text{for } Q \geq 1000. \end{cases}$$

Determine the optimal order quantity.

ANSWER

The cost per annum is

$$K(Q_= \frac{sd}{Q} + \frac{1}{2}hQ + dc$$

First, find the optimal value of Q in the range $0 \leq Q \leq 500$.

$$K(Q) = \frac{sd}{Q} + \frac{1}{2}hQ + d\left[50 - \frac{Q}{100}\right]$$

$\frac{dK(Q)}{dQ} = 0$ gives $\frac{-sd}{Q^2} + \frac{1}{2}h - \frac{d}{100} = 0$, and

$$Q = \sqrt{\frac{2sd}{h - \frac{d}{50}}}$$

Substituting $s = 200,\ d = 2000,\ h = 45$ we obtain

$$Q = \sqrt{\frac{2.200.2000}{45 - 40}} = 400$$

Thus, for $0 \leq Q \leq 500,\ k$ is minimized when $Q = 400$, and

$$K(400) = \frac{200.2000}{400} + \frac{1}{2}.45.400 + 2000(50 - 4) = 102000$$

The ECQ value is $Q + \sqrt{\frac{2sd}{h}} = 133.33$. This shows that for $500 \leq Q < 1000$, and for $Q \geq 1000,\ K(Q)$ is minimized when $Q = 500$; for $Q \geq 1000,\ K(Q)$ is minimized for $Q = 1000$. We already know that $K(500 > K(400)$, so it remains to evaluate $K(1000)$.

$$K(1000) = \frac{200.2000}{1000} + \frac{1}{2}45.1000 + 2000.40 = 102900$$

Thus the optimal order quantity is $Q = 400$.