**Question**

Explain the major difference between an ASIAN option and a LOOKBACK option. Based on general financial arguments, would you expect a Lookback option to more or less valuable than its European vanilla equivalent?
A STOP-LOSS option may be regarded as a perpetual barrier lookback Put which pays a fixed proportion $\lambda$ of the maximum realized asset price $J$ if ever the asset price falls below some barrier which will be denoted by $\lambda J$, $(\lambda < 1)$. Assume that a stop-loss option is written on an underlying that pays a continuous dividend yield $D_0$. By seeking a time-independent solution to the Black-Scholes equations (with suitable boundary conditions) of the form

$$V = JW(S/J)$$

value this option. What is the major financial reason for buying such an option?
What value does the option have when $D_0 = 0$? Give a financial interpretation of this result.

**Answer**
For a lookback option the payoff depends not only on the asset price at Expiry, but also on the min or max of the asset price over some period prior to expiry. They are similar to Asians save for the fact that an Asian depends on an AVERAGE not a max or a min.
Essentially a Lookback allows the holder to sell at the best price of the asset. This is a huge advantage, so the lookback should be more expensive than its Euro Vanilla equivalent.
Now consider a stop-loss option. Since it is a perpetual, $\frac{\partial}{\partial t} = 0$ and $V$ satisfies

$$\frac{1}{2}\sigma^2 S^2 V_{SS} + (r - D_0)SV_S - rV = 0.$$

Now if ever $S$ reaches $\lambda J$, the payoff is $\lambda J$.

$$\Rightarrow V(\lambda J, J) = \lambda J$$

So now try $v = JW(\eta)$, $\quad (\eta = S/J)$ and use dash for $\frac{\partial}{\partial \eta}$.

$$\begin{aligned} \Rightarrow \frac{1}{2}\sigma^2 S^2 W''/J + (r - D_0)SW' - rJW &= 0 \\ \Rightarrow \frac{1}{2}\sigma^2 \eta^2 W'' + (r - D_0)\eta W' - rW &= 0 \end{aligned}$$

(also $V(\lambda J, J) = \lambda J) \rightarrow W(\lambda) = \lambda$.)
Euler's equation:- try $W \propto \eta^\alpha$

$$\Rightarrow \frac{1}{2}\sigma^2 \alpha(\alpha - 1) + (r - D_0)\alpha - r = 0$$

$$\alpha^\pm = \frac{-(r - D_0) + \frac{\sigma^2}{2} \pm \sqrt{\left(r - D_0 - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}}{\sigma^2}$$

We need a final B/C. Once a new max had been reached, we have $V_J = 0$ when $S = J$ since 'old' maxima are forgotten instantly.

$$\begin{aligned} \Rightarrow W = \frac{JS}{J^2}W' &= 0 \\ \Rightarrow W(1) &= W'(1) \end{aligned}$$

Now $W = A\eta^{\alpha^+} + B\eta^{\alpha^-}$.

Now impose the 2 B/C's:-

$$\begin{aligned}
\lambda &= A\lambda^{\alpha^+} + B\lambda^{\alpha^-} \\
A + B &= \alpha^+ A + \alpha^- B \\
\Rightarrow A &= \frac{\lambda(\alpha^- - 1)}{\lambda^{\alpha^+}(\alpha^- - 1) + (1 - \alpha^+)\lambda^{\alpha^-}} \\
B &= \frac{\lambda(1 - \alpha^+)}{\lambda^{\alpha^+}(\alpha^- - 1) + \lambda^{\alpha^-(1-\alpha^+)}}
\end{aligned}$$

$$\Rightarrow W = \lambda\left[\frac{\eta^{\alpha^+}(1 - \alpha^-) - \eta^{\alpha^-}(1 - \alpha^+)}{\lambda^{\alpha^+}(1 - \alpha^-) - \lambda^{\alpha^-}(1 - \alpha^+)}\right]$$

The major financial reason for buying this option is to safeguard one's earlier success against a sudden sharp fall in the asset price.
When $D_0 = 0$

$$\alpha^{\pm} = \frac{\frac{\sigma^2}{2} - r \pm \sqrt{\left(r + \frac{\sigma^2}{2}\right)^2}}{\sigma^2}$$

$\Rightarrow \alpha^+ = 1,\ \alpha^- = -2r/\sigma^2$

$$W = \lambda\left[\frac{\eta(1 + 2r) - \eta^{-\frac{2r}{\sigma^2}}(1 - 1)}{\lambda(1 + 2r) - \eta^{-\frac{2r}{\sigma^2}}(1 - 1)}\right] = \eta$$

Just the asset. Obvious since with no dividends you might just as well buy the asset.