Vector Fields
Scalar and Vector Fields

Question
Describe the streamlines of the following velocity field.
$\underline{v}(x,y) = x\underline{i} + (x+y)\underline{j}$, *Hint: let* $y = xv(x)$.
Answer
The field lines satisfy

$$\begin{aligned} \frac{dx}{x} &= \frac{dy}{x+y} \\ \frac{dy}{dx} = \frac{x+y}{x} \quad \text{Let y = xv(x)} & \\ \Rightarrow \frac{dy}{dx} &= v = +\frac{dv}{dx} \\ \Rightarrow V + x\frac{dv}{dx} &= \frac{x(1+v)}{x} \\ &= 1+v \end{aligned}$$

Thus $\frac{dv}{dx} = \frac{1}{x}$, *and so* $v(x) = \ln|x| + C$.
So the field lines have equations $y = x\ln|x| + Cx$.