Vector Fields
Scalar and Vector Fields

Question

Describe the streamlines of the following velocity field.

$\underline{v}(x, y, z) = y\underline{i} - x\underline{j} + \underline{k}$

Answer

The streamlines satisfy $\frac{dx}{y} = -\frac{dy}{x} = dz$. Thus $xdx + ydy = 0$, so $x^2 + y^2 = C_1^2$.

Therefore

$$\frac{dz}{dx} = \frac{1}{y} = \frac{1}{\sqrt{C_1^2 - x^2}}.$$

This implies that $z = \sin^{-1} \frac{x}{C_1} + C_2$.

The streamlines are the spirals in which the surfaces $x = C_1 \sin(z - C_2)$ intersect the cylinders $x^2 + y^2 = C_1^2$.