QUESTION

(a) Evaluate the double integral $\int_0^2 \int_{y^2}^2 y(2x - y^2/4)\, dxdy$.

(b) Given the integral $\int_0^4 \int_{\sqrt{y}}^2 y \sin x^5\, dxdy$, sketch the region of integration. Reverse the order of integration and evaluate the resulting integral.

(c) Sketch the plane region R defined by the inequalities, computing the intercepts of the defining curves with the x and y axes and with each other:

$$x \leq \sqrt{y+6},\ x \geq \sqrt{2y},\ y \geq 0.$$

By choosing an appropriate order of integration show how to express an integral of the form $\iint_R f(x,y)\, dA$ as a double integral, computing the limits.

ANSWER

(a)

$$\begin{aligned}
\int_0^2 \int_{\sqrt{y}}^2 y(2x - y^2/4)\, dxdy &= \int_0^2 \left[x^2 y - x\frac{y^3}{4} \right]_{y^2}^2 \\
&= \int_0^2 \left[4y - \frac{2y^3}{4} - y^5 + \frac{y^5}{4} \right] dy \\
&= \left[4\frac{y^2}{2} - \frac{y^4}{8} - \frac{y^6}{6} + \frac{y^6}{24} \right]_0^2 0 \\
&= -2
\end{aligned}$$

(b) DIAGRAM

$$\int_0^2 \int_0^{x^2} y \sin x^5\, dydx$$

$$= \int_0^2 \left[\frac{y^2}{2} \sin x^5 \right]_0^{x^2} = \int_0^2 \left[\frac{x^4}{2} \sin x^5 \right] dx$$

put $u = x^5$ so $\frac{du}{dx} = 5x^4$ to get

$$\begin{aligned}\int_0^2 \frac{1}{10}\sin u\,du &= \left[-\frac{1}{10}\cos u\right]_{x=0}^{x=2} \\ &= -\frac{1}{10}\cos 32 + \frac{1}{10}\end{aligned}$$

(c) DIAGRAM

$$\iint_R f(x,y)dA = \int_0^6 \int_{\sqrt{2y}}^{\sqrt{y+6}} f(x,y)\,dxdy$$