

Question

A particle has a path given by $x = \cos \omega t$, $y = \sin \omega t$, $z = t$. Find its velocity and acceleration in cylindrical polar coordinates.

Answer

$$\mathbf{v} = \frac{d}{dt}(\mathbf{i} \cos \omega t + \mathbf{j} \sin \omega t + t\mathbf{k}) = -\omega \sin \omega t \mathbf{i} + \omega \cos \omega t \mathbf{j} + \mathbf{k}$$

$$\mathbf{r} = \mathbf{i} \cos \omega t + \mathbf{j} \sin \omega t + z\mathbf{k}$$

in cylindrical polar coordinates: $r = \sqrt{x^2 + y^2} = 1$; $\phi = \omega t$; $z = t$.

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\phi}\mathbf{e}_\phi + \dot{z}\mathbf{e}_z$$

$$\mathbf{v} = \omega\mathbf{e}_\phi + \mathbf{e}_z$$

$$\mathbf{a} = (\ddot{r} - r\dot{\phi}^2)\mathbf{e}_r + \frac{1}{r}(\dot{r}^2\dot{\phi})\mathbf{e}_\phi + \ddot{z}\mathbf{e}_z$$

$$\mathbf{a} = -\omega^2\mathbf{e}_r$$