

QUESTION

Large batches of components are delivered to two factories  $A$  and  $B$ . Each batch is subjected to an acceptance sampling scheme as follows:

Factory  $A$ : Accept the batch if a random sample of 10 components contains less than 2 defectives. Otherwise reject the batch.

Factory  $B$ : Take a random sample of 5 components. Accept the batch if this sample contains no defectives. Reject the batch if this sample contains 2 or more defectives. If the sample contains 1 defective, take a further sample of 5 and accept the batch if this sample contains no defectives.

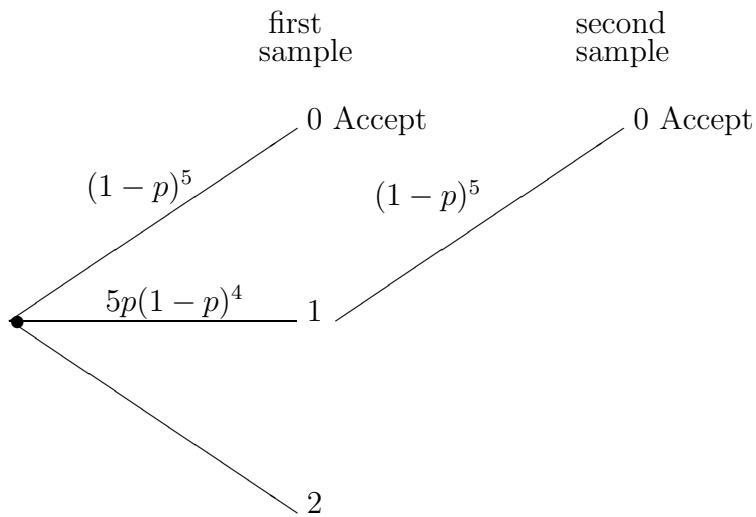
If the fraction defective in the batch is  $p$ , find the probabilities of accepting a batch under each scheme.

Write down an expression for the average number sampled in factory  $B$  and find its maximum value.

ANSWER

$n = 10$ ,  $p$  = proportion defective.

$A$ : accept if less than 2 defectives.  $P(\text{accept}) = q^{10} = 10q^9p = (1-p)^9(1+9p)$



$$P(\text{accept}) = (1-p)^5 + 5p(1-p)^9$$

$$\text{Expected number sampled} = 5 + 5 \times 5p(1-p)^4 = E$$

$$\frac{\partial E}{\partial p} = -25p(1-p)^3 \times 4 + 25(1-p)^4 = 0 \text{ when } 4p = 1-p \quad p = 0.2$$

$$E = 5 + 5 \times 0.8^4 = 7.048 \text{ (check } \frac{\partial^2 E}{\partial p^2} \leq 0 \text{ when } p = 0.2).$$