

QUESTION

A new car of a certain model may be assumed to have X minor faults where X has a Poisson distribution with mean μ . A report is sent to the manufacturer listing the faults for each car which has at least one fault. Write down the probability function of Y , the number of faults listed on a randomly chosen report card and find $E(Y)$. Given $E(Y) = 2.5$ find μ correct to 1 decimal place.

ANSWER

$$X \sim P(\mu) \quad p(x) = e^{-\mu} \frac{\mu^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$P(X = 0) = p(0) = e^{-\mu} \quad P(X \neq 0) = 1 - e^{-\mu}$$

$$\begin{aligned} P(Y = y) &= P(X = y | X \neq 0) \\ &= \frac{P(X = y \text{ and } X \neq 0)}{P(X \neq 0)} \\ &= \frac{P(X = y)}{P(X \neq 0)} \\ &= \frac{e^{-\mu} \mu^y}{y!(1 - e^{-\mu})}, \quad y = 1, 2, \dots \end{aligned}$$

$$\begin{aligned} E(Y) &= \sum_{y=1}^{\infty} \frac{y e^{-\mu} \mu^y}{y!(1 - e^{-\mu})} \\ &= \frac{1}{1 - e^{-\mu}} \sum_{y=0}^{\infty} \frac{e^{-\mu} \mu^y y}{y!} \\ &= \frac{\mu}{1 - e^{-\mu}} \end{aligned}$$

Since the summation is $E(x)$. Given $E(Y) = 2.5 = \frac{\mu}{1 - e^{-\mu}}$, we know that $\mu < 2.5$ and we can use trial and error to obtain $\mu \approx 2.2$