QUESTION

In each of the following cases write down the probability function for the given variable and calculate its mean and variance.

(i) the score on an unbiased die,

(ii) the sum of the scores on two unbiased dice,

(iii) the maximum score on one die when two dice are tossed,

(iv) the score on a die which is biased so that the probability that it lands with any face uppermost is proportional to the number on that face.

In case (ii) comment on your results.

ANSWER

(i)

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\mu = 3.5$

$$\begin{aligned}\sigma^2 &= \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) - 3.5^2\\ &= \frac{1}{6}6713 - \frac{49}{4} = \frac{35}{12}\end{aligned}$$

(ii)

		first					
	sum	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
second	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Each probability is $\frac{1}{36}$ since the two dice are independent.

x	2	3	4	5	6	7	8	9	10	11	12
p(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\mu = 7$ (calculate or symmetry)

$\sigma^2 = \frac{1}{36}(2^2 + 23^2 + 34^2 + \ldots + 112^2) - 49 = \frac{1974}{36} - 49 = \frac{35}{6}$

$\mu = 7 = 2 \times 3.5 = 2 \times \mu$ from (i)

$\sigma^2 = \frac{35}{6} = 2 \times \frac{35}{12} = 2 \times \sigma^2$ from (i)

(iii)

	max	first 1	2	3	4	5	6
	1	1	2	3	4	5	6
	2	2	2	3	4	5	6
	3	3	3	3	4	5	6
second	4	4	4	4	4	5	6
	5	5	5	5	5	5	6
	6	6	6	6	6	6	6

Each probability is $\frac{1}{36}$ since the dice are independent.

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$\mu = \frac{1}{36}(1 \times 1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + 5 \times 9 + 11 \times 36) = \frac{161}{36} = 4.472$

$\sigma^2 = \frac{1}{36}(1^2 \times 1 + 2^2 \times 3 + 3^2 \times 5 + 4^2 \times 7 + 5^2 \times 9 + 11^2 \times 36) - \mu^2 = 1.971$

(iv)

x	1	2	3	4	5	6
$p(x)$	k	$2k$	$3k$	$4k$	$5k$	$6k$

$k + 2k + 3k + \ldots 6k = 21k = 1$ therefore $k = \frac{1}{21}$

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

$$\begin{aligned}\mu &= \frac{1}{21}(1 \times 1 + 2 \times 2 + \ldots + 6 \times 6) \\ &= \frac{1}{21}\frac{7 \times 7 \times 13}{6} = \frac{13}{3} = 4.333\end{aligned}$$

$$
\begin{aligned}
\sigma^2 &= \frac{1}{21}(1^2 \times 1 + 2^2 \times 2 + \ldots + 6^2 \times 6) - (\frac{13}{3})^2 \\
&= \frac{1}{21}(\frac{7 \times 7 \times 7}{2})^2 - \frac{169}{9} = \frac{20}{9} = 2.222
\end{aligned}
$$