Exam Question
Topic: Surface of Revolution
Find the area of the curved surface obtained by rotating the graph of $y = \sqrt{x}$,
between $x = 0$ and $x = 1$, about the line $y = 0$.

Solution

$$
\begin{aligned}
\text{Area} \quad &= \quad 2\pi \int_0^1 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = 2\pi \int_0^1 \sqrt{x} \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2} \, dx \\
&= \quad 2\pi \int_0^1 \sqrt{1 + \tfrac{1}{4}} \, dx = \tfrac{4\pi}{3} \left[\left(x + \tfrac{1}{4}\right)^{3/2} \right]_0^1 \\
&= \quad \tfrac{4\pi}{3} \left[\left(\tfrac{5}{4}\right)^{3/2} - \left(\tfrac{1}{4}\right)^{3/2} \right] = \tfrac{\pi}{6} \left(5\sqrt{5} - 1 \right).
\end{aligned}
$$