

### Exam Question

#### Topic: Volume of Revolution

Let  $R$  denote the region in the first quadrant of the  $x$ - $y$  plane bounded by the  $x$ -axis, the  $y$ -axis, and the curve  $y = 1 - x^2$ .

- (i) Find the volume of the solid obtained by rotating  $R$  around the line  $x = 3$ .
- (ii) Find the total surface area of the solid obtained by rotating  $R$  around the  $y$ -axis.

#### Solution

$$\begin{aligned} \text{(i) } V &= 2\pi \int_0^1 (3-x)(1-x^2) dx = 2\pi \int_0^1 (3-x-3x^2+x^3) dx \\ &= 2\pi \left( 3 - \frac{1}{2} - 1 + \frac{1}{4} \right) = \frac{7\pi}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii) } S &= 2\pi \int_0^1 x\sqrt{1+4x^2} dx + \pi \\ &= 2\pi \left[ \frac{1}{12} (1+4x^2)^{3/2} \right]_0^1 + \pi = \frac{\pi}{6} (5^{3/2} - 1) + \pi \\ &= \frac{\pi}{6} (5^{3/2} + 5) = \frac{5\pi}{6} (\sqrt{5} + 1) \end{aligned}$$