Exam Question

Topic: Volume of Revolution

Find the volume of revolution obtained by rotating the region in the x-y plane bounded by the lines $x = 0, y = 0, x = 1$ and the curve $y = e^x$ about (i) the line $y = -2$, (ii) the line $x = 3$.

Give your answer in terms of e, and also as an approximation correct to 3 decimal places, using your calculator.

Solution

(i)

$$
\begin{aligned}
V &= \pi \int_0^1 [(e^x + 2)^2 - 2^2]\, dx = \pi \int_0^1 (e^{2x} + 4e^x)\, dx \\
&= \pi \left[\frac{e^{2x}}{2} + 4e^x \right]_0^1 = \pi \left[\frac{e^2}{2} + 4e - \frac{1}{2} - 4 \right] \\
&= \pi \left[\frac{e^2}{2} + 4e - \frac{9}{2} \right] = 31.628 \ (3 \ \text{d.p.})
\end{aligned}
$$

(ii)

$$
\begin{aligned}
V &= 2\pi \int_0^1 (3 - x)e^x\, dx = 2\pi \left[(3 - x)e^x \right]_0^1 + 2\pi \int_0^1 e^x\, dx \\
&= 2\pi(2e - 3) + 2\pi(e - 1) = 2\pi(3e - 4) = 26.106 \ (3 \ \text{d.p.})
\end{aligned}
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