

### Exam Question

#### Topic: Volume of Revolution

Find the volume of revolution obtained by rotating the region in the  $x$ - $y$  plane bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x = 1$  and the curve  $y = e^x$  about

(i) the line  $y = 3$ , (ii) the line  $x = -2$ .

Give your answer in terms of  $e$ , and also as an approximation correct to 3 decimal places, using your calculator.

#### Solution

(i)

$$\begin{aligned} V &= \pi \int_0^1 [3^2 - (3 - e^x)^2] dx = \pi \int_0^1 (6e^x - e^{2x}) dx \\ &= \pi [6e^x - e^{2x}/2]_0^1 = \pi [6e - e^2/2 - 6 + 1/2] \\ &= \frac{\pi}{2} [12e + e^2 - 11] = 22.353 \text{ (3 d.p.)} \end{aligned}$$

(ii)

$$\begin{aligned} V &= 2\pi \int_0^1 (x+2)e^x dx = [2\pi(x+2)e^x]_0^1 - \int_0^1 2\pi e^x dx \\ &= [2\pi(x+2)e^x]_0^1 - [2\pi e^x]_0^1 = 2\pi(2e - 1) = 27.867 \text{ (3 d.p.)} \end{aligned}$$