Exam Question
Topic: Volume of Revolution
Find the volume of revolution obtained by rotating the region in the x-y plane bounded by the lines $x = 0, y = 0, x = 1$ and the curve $y = e^x$ about (i) the line $y = 3$, (ii) the line $x = -2$.
Give your answer in terms of e, and also as an approximation correct to 3 decimal places, using your calculator.

Solution
(i)

$$
\begin{aligned}
V &= \pi \int_0^1 [3^2 - (3 - e^x)^2] \, dx = \pi \int_0^1 (6e^x - e^{2x}) \, dx \\
&= \pi \left[6e^x - e^{2x}/2 \right]_0^1 = \pi[6e - e^2/2 - 6 + 1/2] \\
&= \frac{\pi}{2}[12e + e^2 - 11] = 22.353 \ (3 \text{ d.p.})
\end{aligned}
$$

(ii)

$$
\begin{aligned}
V &= 2\pi \int_0^1 (x + 2)e^x \, dx = [2\pi(x + 2)e^x]_0^1 - \int_0^1 2\pi e^x \, dx \\
&= [2\pi(x + 2)e^x]_0^1 - [2\pi e^x]_0^1 = 2\pi(2e - 1) = 27.867 \ (3 \text{ d.p.})
\end{aligned}
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