

QUESTION Find all solutions (if any) of each of the following systems of equations:

(a)

$$\begin{aligned}3x + 6y - 6z &= 9 \\2x - 5y + 4z &= 6 \\-x + 16y - 14z &= -3\end{aligned}$$

(b)

$$\begin{aligned}x + y - z &= 7 \\4x - y + 5z &= 4 \\6x + y + 3z &= 20\end{aligned}$$

(c)

$$\begin{aligned}2w + x + 2y - z &= 6 \\6w + 8x + 12y - 13z &= -21 \\10w + 2x + 2y + 3z &= 59 \\-4w + y - 3z &= -30\end{aligned}$$

ANSWER

(a) Gaussian elimination leads to  $\left[ \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 9 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$$\begin{array}{l} \text{So} \quad x = 3 + \frac{2z}{9} \quad \text{or} \quad x = 3 + \frac{y}{4} \quad \text{or} \quad x = 3 + 2\lambda \\ \quad \quad y = \frac{8z}{9} \quad \quad \quad y = y \quad \quad \quad y = 8\lambda \\ \quad \quad z = z \quad \quad \quad z = \frac{9y}{8} \quad \quad \quad z = 9\lambda \end{array}$$

(b) Add twice row 1 to row 2, to get  $6x + y + 3z = 18$ , and this is clearly inconsistent with row 3, so there are no solutions.

(c)  $w = 4, x = 1, y = 1, z = 5$ .