

## Exam Question

### Topic: TripleIntegral

A hemispherical solid of radius  $a$  is resting on its circular face on a horizontal plane. The density at any point of the solid is proportional to the square of the distance of the point from the horizontal plane. Calculate the mass of the object and its mean density.

### Solution

Density =  $kz^2$  in cartesian coordinates. We perform the integration in spherical polars.

$$\begin{aligned} M &= \int_0^a dr \int_0^{2\pi} d\phi \int_0^{\pi/2} kr^2 \cos^2 \theta r^2 \sin \theta d\theta \\ &= k \int_0^a r^4 dr \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta \\ &= \frac{ka^5}{5} 2\pi \left[ -\frac{\cos^3 \theta}{3} \right]_0^{\pi/2} = \frac{2\pi k^5}{15} \end{aligned}$$