

Exam Question

Topic: TripleIntegral

Let C denote a solid cylinder of height 2 and radius 1 whose axis of symmetry is the z -axis. The density of this cylinder at a point P is equal to the product of the distance of P from the bottom of the cylinder and the distance of P from the z -axis. Find the total mass of the cylinder C by evaluating an appropriate triple integral.

Solution

In cylindrical polar coordinates the density is rz . So the mass is given by

$$\int_{\phi=0}^{\pi} d\phi \int_{z=0}^2 dz \int_{r=0}^1 rz \cdot r dr = 2\pi \int_0^2 z dz \int_0^1 r^2 dr = 2\pi \times 2 \times \frac{1}{3} = \frac{4\pi}{3}$$