

Question

Show that the speed of a planet in a planetary orbit satisfies

$$v^2 = \frac{\mu}{m} \left\{ \frac{2}{r} - \frac{1}{a} \right\}.$$

Answer

Use the energy equation $\frac{1}{2}mv^2 - \frac{\mu}{r} = E$

But $a = -\frac{\mu}{2E}$

Therefore $\frac{1}{2}mv^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \Rightarrow v^2 = \frac{\mu}{m} \left(\frac{2}{r} - \frac{1}{a} \right)$