Exam Question
Topic: SurfaceIntegral
Let S be part of the curved surface of a cylinder, specified by

$$y^2 + z^2 = 1,\ z > 0,\ 0 \le x \le 1.$$

Evaluate the surface integral

$$\iint_S \left(\sin(xyz)\mathbf{i} + z\mathbf{j} + y^2\mathbf{k}\right) \cdot \mathbf{dS}.$$

Solution
The equation of the cylinder is $F(x, y, z) = y^2 + z^2 - 1 = 0$.
Differentiating gives $\dfrac{\partial F}{\partial x} = 0;\ \ \dfrac{\partial F}{\partial y} = 2y;\ \ \dfrac{\partial F}{\partial z} = 2z.$
So $\mathbf{dS} = \dfrac{2y\mathbf{j} + 2z\mathbf{k}}{|2z|}\, dxdy = \left(\dfrac{y}{z}\mathbf{j} + \mathbf{k}\right) dxdy$ as $z > 0$.
So $(\sin(xyz)\mathbf{i} + z\mathbf{j} + y^2\mathbf{k}) \cdot \mathbf{dS} = (y + y^2)dxdy$.
The surface integral then becomes

$$\int_0^1 dx \int_{-1}^1 (y + y^2)\, dy = \frac{2}{3}.$$