**Question**
Show that the elliptic integral

$$I(m) = \int_0^{\frac{\pi}{2}} d\theta\ \sqrt{1 - m^2 \sin^2\theta}$$

has the expansion

$$I(m) = \frac{\pi}{2}\left(1 - \frac{1}{4}m - \frac{3}{64}m^2 - \frac{5}{256}m3 - \frac{175}{16384}m^4 + \cdots\right)$$

as $m \to 0$.

$$\textbf{Hint}: \int_0^{\frac{\pi}{2}} \sin 2\mathrm{n}\theta\,\theta = \pi\frac{(2\mathrm{n})!}{\{2^{\mathrm{n}+1}\mathrm{n}!\}}$$

**Answer**
Do it the dirty way! $m$ is small so a binomial expansion may work!
$\sqrt{1 - m^2\sin^2\theta} = 1 - \dfrac{1}{2}\dfrac{m^2\sin^2\theta}{1!} + \dfrac{(\frac{1}{2})(-\frac{1}{2})}{2!}m^4\sin^4\theta + \cdots$
Need $m^2\sin^2\theta < 1$ for convergence.
So if $O < \theta < \dfrac{\pi}{2} \Rightarrow 0 < \sin\theta < 1$ this last bit means $|m^2| < 1$ to be safe.
General expansion is:
$\sqrt{1 - m^2\sin^2\theta} = 1 - \sum_{n=1}^{\infty} \dfrac{m^2\sin^{2n}\theta}{n!} \quad m^2 > 1$
so integrating term by term $\displaystyle\int_0^{\frac{\pi}{2}} d\theta$ we get from the hint in the question

$$I(m) = \frac{\pi}{2}\left(1 - \frac{1}{4}m - \frac{3}{64}m^2 - \frac{5}{256}m^3 - \frac{175}{16384}m^4 + \cdots\right) \text{ etc.}$$