QUESTION
Two shares follow geometric Brownian motions, i.e.,

$$ds_1 = S_1(\mu_1 dt + \sigma_1 dW_1)$$

$$ds_2 0 S_2(\mu_2 dt + \sigma_2 dW_2)$$

The price changes are correlated with the function coefficient ρ, i.e. $dW_1 dW_2 = \rho dt$. Starting from Taylor'S theorem extended to stochastic variables, find the stochastic differential equation satisfied by a function $f(S_1, S_2)$.
ANSWER
Taylor for 2 variables,

$$df = \frac{\partial f}{\partial S_1} ds_1 + \frac{\partial f}{\partial S_2} ds_2 + \frac{1}{2}\frac{\partial^2 f}{\partial S_1^2}(ds_1)^2 + \frac{\partial^2 f}{\partial S_1 \partial S_2} ds_1 ds_2 + \frac{\partial^2 f}{\partial S_2}(ds_2)^2 + \dots$$

(no explicit t dependence). But $ds_i = S_i(\mu_i dt + \sigma_i dw_i)$

$$\begin{aligned}
\Rightarrow df &= \frac{\partial f}{\partial S_1} S_1(\mu_1 dt + \sigma_1 dw_1) + \frac{\partial f}{\partial S_2} S_2(\mu_2 dt + \sigma_2 dw_2) + \\
&\quad \frac{1}{2}\frac{\partial^2 f}{\partial S_1^2} S_1^2(\mu_1 dt + \sigma_1 dw_1)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial S_2} S_2^2(\mu_2 dt + \sigma_2 dw_2)^2 + \\
&\quad \frac{\partial^2 f}{\partial S_1 \partial S_2} S_1 S_1(\mu_1 dt + \sigma_1 dw_1)(\mu_2 dt + \sigma_2 dw_2) \\
&= \left(\frac{\partial f}{\partial S_1} S_1 \mu_1 + \frac{\partial f}{\partial S_2} S_2 \mu_2\right) dt + \frac{\partial f}{\partial S_1} S_1 \sigma_1 dw_1 + \frac{\partial f}{\partial S_2} S_2 \sigma_2 dw_2 \\
&\quad + \frac{1}{2}\frac{\partial^2 f}{\partial S_1^2} S_1^2 \left(\mu_1^2 (dt)^2 + 2\mu_1 \sigma_1 dt dw_1 + \sigma_1^2 (dw_1)^2\right) \\
&\quad + \frac{1}{2}\frac{\partial^2 f}{\partial S_2^2} S_2^2 \left(\mu_2^2 (dt)^2 + 2\mu_2 \sigma_2 dt dw_2 + \sigma_1^2 (dw_2)^2\right) \\
&\quad + \frac{\partial^2 f}{\partial S_1 \partial S_2} S_1 S_2 \left(\mu_1 \mu_2 dt^2 + \mu_1 dt dw_2 \sigma_2 + \mu_2 \sigma_1 dt dw_1 + \sigma_1 \sigma_2 dw_1 dw_2\right)
\end{aligned}$$

Now, eliminate the smaller terms: $(dt)^2 = 0$, $dt dw_1 = 0$, $dt dw_2 = 0$, $(dw_1)^2 = dt$, $(dw_2)^2 = dt$ (since w_1, w_2 are both $\in N(0,t)$) and $dw_1 dw_2 = \rho dt$ as per hint.
Collecting like terms

$$
\begin{aligned}
df \quad = \quad & \left(\frac{\partial f}{\partial S_1} S_1 \mu_1 + \frac{\partial f}{\partial S_2} S_2 \mu_2 + \frac{1}{2} S_1^2 \sigma_1^2 \frac{\partial^2 f}{\partial S_1^2} + \frac{\partial^2 f}{\partial S_1 \partial S_2} S_1 S_2 \sigma_1 \sigma_2 + \frac{1}{2} S_2 62 + \sigma_2^2 \frac{\partial^2 f}{\partial S_2^2} \right) dt \\
& + \left(\frac{\partial f}{\partial S_1} S_1 \sigma_1 dw_1 + \frac{\partial f}{\partial S_2} S_2 \sigma_2 dw_2 \right)
\end{aligned}
$$