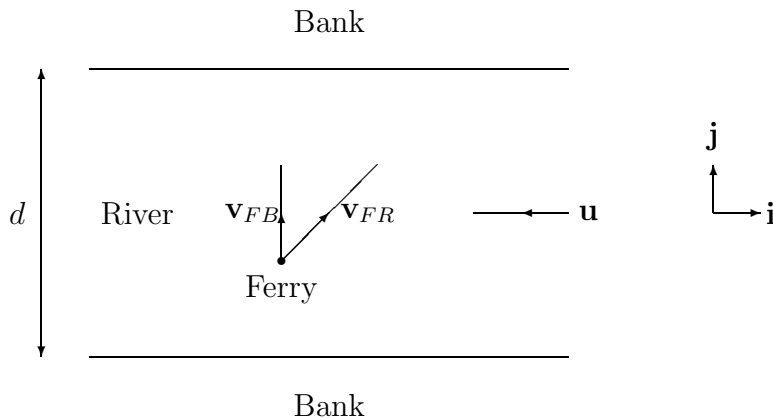


### Question

A river of width  $d$  has uniform current with speed  $u$ . If the ferry can make speed  $v$  relative to the water at what angle should the ferry set off from one bank in order to arrive directly opposite on the far bank?

How long does it take the ferry to cross the river?

### Answer



Use the equation  $\mathbf{v}_{FB} = \mathbf{v}_{FR} + \mathbf{v}_R$ ;

$\mathbf{v}_{FB}$  is the velocity of the ferry relative to the bank

where  $\mathbf{v}_{FR}$  is the velocity of the ferry relative to the river

$\mathbf{v}_R$  is the velocity of the river relative to the bank

$$\mathbf{v}_{FB} = v_F \mathbf{j}; \quad \mathbf{v}_R = -u \mathbf{i}; \quad \mathbf{v}_{FR} = v(\mathbf{i} \cos \theta + \mathbf{j} \sin \theta)$$

$$\Rightarrow \mathbf{v}_{FB} = v(\mathbf{i} \cos \theta + \mathbf{j} \sin \theta) - u \mathbf{i}$$

$$\Rightarrow 0 = \mathbf{i}(v \cos \theta - u) + \mathbf{j}(v \sin \theta - v_{FB})$$

$$\Rightarrow \theta = \cos^{-1} \frac{u}{v}$$

$$\text{and } v_{FB} = v \sin \theta = v \sqrt{1 - \frac{u^2}{v^2}} = \sqrt{v^2 - u^2}$$

Time to cross the river is  $\frac{d}{v_{FB}} = \frac{d}{\sqrt{v^2 - u^2}}$