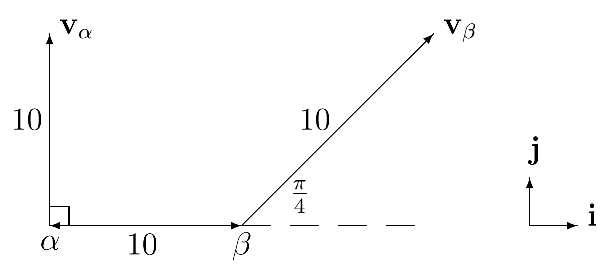
Question

The ship HMS Alpha is steaming due north with a speed 10kmh^{-1}, whilst HMS Beta is steaming north east with the same speed. Find the velocity of HMS Alpha relative to HMS Beta. If at noon HMS is positioned 10km due west of HMS Beta show that subsequently the position of HMS Alpha relative to HMS Beta is $10\left[-\left(1+\frac{t}{\sqrt{2}}\right)\mathbf{i}+\left(1-\frac{1}{\sqrt{2}}\right)t\mathbf{j}\right]$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors oriented east and north and t is the time elapsed past noon. At what time was HMS Alpha due south of HMS Beta?

Answer

Situation at noon:



$$\mathbf{v}_{\alpha\beta} = \mathbf{v}_\alpha - \mathbf{v}_\beta = 10\mathbf{j} - \frac{10}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) = 10\left(-\frac{1}{\sqrt{2}}\mathbf{i} + \left(1-\frac{1}{\sqrt{2}}\right)\mathbf{j}\right)$$

Position of α relative to β satisfies $\frac{d}{dt}\mathbf{r}_{\alpha\beta} = \mathbf{v}_{\alpha\beta}$

Thus $\mathbf{r}_{\alpha\beta} = \mathbf{v}_{\alpha\beta}t + \mathbf{a}$ where $\mathbf{a}$ is a constant of integration.

At t = 0, $\mathbf{r}_{\alpha\beta} = -10\mathbf{i}$ (position of α relative to β at noon)

$$\begin{aligned}\mathbf{r}_{\alpha\beta} &= \frac{10}{\sqrt{2}}[-\mathbf{i} + (\sqrt{2}-1)\mathbf{j}]t - 10\mathbf{i} \\ &= \frac{10}{\sqrt{2}}[-\mathbf{i}(t+\sqrt{2}) + t(\sqrt{2}-1)\mathbf{j}]\end{aligned}$$

α is due south of β when $\mathbf{r}_{\alpha\beta}$ has no $\mathbf{i}$ component

i.e. at t = -$\sqrt{2}$hrs ≈ -1.414hrs or at 10.35 am

The distance is $\frac{10}{\sqrt{2}}\sqrt{2}(\sqrt{2}-1) = 414$km